Effects of Shadowing in
Double Pomeron Exchange Processes

E. Gotsman[1]

E.M. Levin [2),3),a]

U. Maor [1),3]

[1] School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv 69978, Israel

[2] Mortimer and Raymond Sackler Institute of Advanced Studies
School of Physics and Astronomy, Tel Aviv University
Tel Aviv, 69978, Israel

[3] LAFEX, Centro Brasileiro de Pesquisas Físicas / CNPq
Rua Dr. Xavier Sigaud 150, 22290 - 180, Rio de Janiero, RJ, Brasil

Abstract: The effects of shadowing in double Pomeron exchange processes are investigated within an eikonal approach with a Gaussian input. Damping factors due to screening are calculated for this process and compared with the factors obtained for total, elastic and single diffraction cross sections. Our main conclusion is that counting rate calculations, of various double Pomeron exchange processes (without screening corrections) such as heavy quark and Higgs production are reduced by a factor of 5 in the LHC energy range, when screening corrections are applied.

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a) On leave from Petersburg Nuclear Institute, 188350, Gatchina, St. Petersburg, Russia

The process of double Pomeron exchange (DPE), shown in Fig.1, has been recognized for sometime [1,2] as an interesting window through which we can further persue our study of Pomeron dynamics, and extend our knowledge of diffraction. Even though DPE processes have relatively small cross sections, they have a very clean experimental signature, where the central diffractive cluster is seperated from the remnants of the two projectiles by large rapidity gaps. (For a schematic lego plot see Fig.2). DPE processes have recently attracted a considerable amount of attention as a possible background for rare electroweak events [3], as well as actual sources for central diffraction of $q\bar{q}$ jets [4] and minijets [5], heavy flavor production [6], Higgs production [7] and an interesting configuration for the study of the Pomeron structure [8].

In this paper we wish to examine the consequences of including screening corrections in the initial state of DPE diagrams. Our calculations are applicable to DPE calculated either in the conventional Regge formalism, or through a two gluon exchange approximation [9, 10]. We show that these s-channel unitarity corrections, cause DPE processes to be strongly suppressed throughout the Tevatron energy range, and even more so at higher energies. The degree of this suppression can easily be assessed in terms of a damping factor $< |D|^2 >$. In this paper we proceed to calculate the damping factor for DPE processes, and make a realistic evaluation of some of the associated final states. As we shall show, our estimates are considerably smaller than the uncorrected rates, published previously.

The present investigation extends our ongoing study [11] on the implementation of s-channel unitarity corrections to Pomeron exchange in high energy hadron scattering. Our study has mostly concentrated on a supercritical DL soft Pomeron [12]

$$\alpha(t) = 1 + \Delta + \alpha' t$$

with $\Delta \simeq 0.085$ and $\alpha' \simeq 0.25 \ GeV^{-2}$. This simple model is rather successful in reproducing the available hadronic data on total and elastic cross sections. We note, nevertheless, that our method is equally effective if we choose to calculate with the hard BFKL QCD-Pomeron [13]. This will be the main subject of a paper to be published shortly.

In previous publications [11] we have attempted a systematic study of s-channel unitarity screening corrections in an eikonal approximation, where our b-space amplitude is written as

$$a(s, b) = i(1 - e^{-\Omega(s,b)}) \tag{1}$$

To obtain analytic expressions for the cross sections of interest, we make the following simplifying assumptions:

1) The opacity $\Omega(s, b)$ is a real function, i.e. a(s,b) is pure imaginary, when necessary analyticity and crossing symmetry can be easily restored [11].

2) We assume our input opacity to be a Gaussian

$$\Omega(s,b) = \nu(s)e^{-\frac{b^2}{R^2(s)}} \tag{2}$$

which corresponds to an exponential behaviour of the input amplitude in t-space. For a supercritical amplitude we have

$$\nu(s) = \frac{\sigma_0}{2\pi R^2(s)}(\frac{s}{s_0})^\Delta \tag{3}$$

where

$$R^2(s) = 4[R_0^2 + \alpha' ln\frac{s}{s_0}] \tag{4}$$

and $\sigma_0 = \sigma(s_0)$.

3) Our eikonal approximation does not include diffractive rescattering. Neglecting these states is a reasonable approximation, as $\frac{\sigma_{diff}}{\sigma_{inel}} \ll 1$ throughout the energy domain of interest.

The simplified model, having the properties described above, reproduces the main features of the elastic and diffractive channels under investigation. To obtain an estimate of the suppression induced by s-channel unitarity screening corrections, we define a damping factor $< |D_i|^2 >$, which is the ratio of the eikonalized output cross section of interest to the uncorrected input cross section. From the definitions of σ_{tot} and σ_{el} [11] we have

$$< |D_{tot}|^2 >= \frac{\sigma_{tot}^{out}}{\sigma_{tot}^{in}} = 1 - \sum_{n=1}^{\infty}(-1)^{n+1}\frac{\nu^n}{(n+1)^2 n!} \tag{5}$$

$$< |D_{el}|^2 >= \frac{\sigma_{el}^{out}}{\sigma_{el}^{in}} = 1 - 4\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\nu^n[2^{n+1}-1]}{(n+2)^2(n+1)!} \tag{6}$$

For the inelastic channels, the damping factor is defined

$$< |D_i|^2 >= \frac{\int d^2b\ a_i(s,b)\ P(s,b)}{\int d^2b\ a_i(s,b)} \tag{7}$$

where $a_i(s,b)$ is the b-space amplitude of interest, and $P(s,b) = e^{-2\Omega(s,b)}$ denotes the probability [11] that no inelastic interaction takes place at impact parameter b. We note that the definition of $< |D_i^2| >$ is correlated to the definition of the survival probability $< |S|^2 >$ in the case of hard parton scattering [3].

We would like to mention that the physical meaning of the damping factor in the parton approach, is the probability to have one parton shower collision in the hadron-hadron interaction. Bjorken's survival probability is the damping factor multiplied by

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the ratio of the input cross section to the inclusive one, with the same trigger in the one parton shower collision. For example the Bjorken survival probability for high p_T jet central diffraction is

$$< |S|^2 > = < |D|^2 > \cdot \frac{\sigma^{one\ parton\ shower}(high\ p_T\ jet\ in\ double\ pomeron\ collision)}{\sigma_{inclusive}(high\ p_T\ jet)} \tag{8}$$

For many reactions the second factor has been calculated, for some even the denominator has been measured. This is the reason for our interest in calculating the damping factor.

As we have seen [11], the damping factor for single diffractive dissociation (SD), (calculated in the triple Pomeron limit), is

$$< |D_{SD}|^2 > = \frac{(M^2 \frac{d\sigma}{dM^2})^{out}}{(M^2 \frac{d\sigma}{dM^2})^{in}} = a(\frac{1}{2\nu})^a \gamma(a, 2\nu) = 1 - a \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2\nu)^n}{(a+n)n!} \tag{9}$$

where

$$\frac{4}{3} < a(s, M^2) = \frac{2R^2(s)}{\bar{R}^2(\frac{s}{M^2}) + 2\bar{R}^2(\frac{M^2}{s_0})} \le 2 \tag{10}$$

and

$$\bar{R}^2(\frac{s}{M^2}) = 2R_0^2 + r_0^2 + 4\alpha' ln(\frac{s}{M^2}) \tag{11}$$

$r_0 \ll R_0^2$ denotes the radius of the triple vertex and can be neglected.
$\gamma(a, 2\nu) = \int_0^{2\nu} z^{a-1} e^{-z} dz$, denotes the incomplete Euler gamma function. One has to integrate over M^2 to obtain the integrated SD cross section σ_{SD}. We find that $a(s, M^2)$ has a rather weak dependence on M^2 as it is proportional to $\alpha' ln M^2$ (with α' small) over a relatively narrow domain. So in practice, one can factor out $< |D_{SD}|^2 >$ in the M^2 integration. Thus we have

$$\frac{\sigma_{SD}^{out}}{\sigma_{SD}^{in}} \simeq < |D_{SD}|^2 > \tag{12}$$

In the exceedingly high energy limit $\nu(s) \gg 1$ and $a(s, M^2) \to 2$ from below. In this limit the screened cross sections differ drastically from the pole input as is evident from Table I. However, in the energy range that is of interest, i.e. HERA-Tevatron-LHC, $\nu(s)$ appears to be of the order of unity (from estimates of screening effect from the available experimental data). For a more realistic estimate we list in Table II the different damping factors at some typical accelerator energies. The calculation is based on a DL input [12] of $\Delta = 0.085$, $\alpha' = 0.25\ GeV^{-2}$ and $R_0^2 = 5.2\ GeV^{-2}$.

We note that the screening corrections saturate at different energy scales for the different channels. In particular, diffractive channels, such as $p\ p \to p\ X$ and even more so

$\gamma\, p \to \psi\, X$, for which $a(s) \to 2$ precociously, exhibit a very tempered energy dependence, which is the result of the early saturation of the screening corrections. This behaviour is to be contrasted with the effective power behaviour of the total and elastic cross sections at these energies.

We now turn to a detailed calculation of central diffraction (CD), which proceeds through DPE. This process was originally calculated by Streng [2]. We follow this calculation and then proceed to calculate $< |D_{CD}|^2 >$. The relevant kinematics are shown in Fig.3. We remind the reader that the t-space elastic scattering amplitude is given by

$$F(s,t) = \frac{1}{2}\nu(s)R^2(s)e^{B(s)t} \tag{13}$$

where the slope of the amplitude $B(s) = \frac{R^2}{4}$. Adopting Streng's notation [2] we have

$$F(s,t) = 2g_P^2(0)(\frac{s}{s_0})^\Delta e^{B(s)t} \tag{14}$$

$$\nu(s) = \frac{4g_P^2(0)}{R^2(s)}(\frac{s}{s_0})^\Delta \tag{15}$$

In this notation

$$\sigma_{tot} = 4\pi F(s,0) = 8\pi g_P^2(0) \tag{16}$$

The cross section of interest is given by

$$
\begin{aligned}
s_1 s_2 \frac{d^4\sigma}{ds_1 ds_2 dt_1 dt_2} &= 4\pi^2 \frac{\sigma_{PP}(M^2)}{\sigma_0^2} \cdot F[\frac{s}{s_1},(\frac{q}{2}+k_1)^2] \cdot F[\frac{s}{s_1},(\frac{q}{2}-k_1)^2] \cdot \\
&\quad F[\frac{s}{s_2},(\frac{q}{2}+k_2)^2] \cdot F[\frac{s}{s_2},(\frac{q}{2}-k_2)^2]
\end{aligned}
\tag{17}
$$

where we have used Streng's definition of σ_{PP}. Eq. (17) can be rewritten as $(t = -q^2)$

$$
\begin{aligned}
s_1 s_2 \frac{d^4\sigma}{ds_1 ds_2 dt_1 dt_2} &= 4\pi^2 \frac{\sigma_{PP}(M^2)}{\sigma_0^2} \cdot (\frac{1}{4})^4 \bar{R}^4(\frac{s}{s_1})\bar{R}^4(\frac{s}{s_2})\bar{\nu}^2(\frac{s}{s_1})\bar{\nu}^2(\frac{s}{s_2}) \cdot \\
&\quad e^{-\bar{B}(\frac{s}{s_1})(2k_1^2+\frac{q^2}{2})} e^{-\bar{B}(\frac{s}{s_2})(2k_2^2+\frac{q^2}{2})}
\end{aligned}
\tag{18}
$$

where

$$\bar{B}(\frac{s}{s_i}) = \frac{1}{2}R_0^2 + \alpha'ln(\frac{s}{s_i}) = \frac{\bar{R}_i^2(\frac{s}{s_i})}{4} \tag{19}$$

$$\bar{\nu}(\frac{s}{s_i}) = \frac{\sigma_0}{2\pi\bar{R}(\frac{s}{s_i})}(\frac{s}{s_i})^\Delta \tag{20}$$

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Note that in Eq.(18) we have a factor 4 in the denominator. This is in accord with the Reggeon calculus rules for identical particles. After integrating over k_1^2 and k_2^2 we have

$$s_1 s_2 \frac{d^2\sigma}{ds_1 ds_2} = \frac{4\sigma_{PP}(M^2)}{\bar{R}^2(\frac{s}{s_1})\bar{R}^2(\frac{s}{s_2})} \cdot g_P^4(0)(\frac{s}{M^2})^{2\Delta} \cdot e^{-\frac{1}{2}(R_0^2 + \alpha' ln\frac{s}{M^2})q^2} \tag{21}$$

Since $M^2 < s_1 < s$ and $s_2 = \frac{M^2 s}{s_1}$ we can integrate over s_1 and obtain

$$M^2 \frac{d\sigma}{dM^2} = \frac{1}{2\alpha'}\sigma_{PP}(M^2)g_0^4(\frac{s}{M^2})^{2\Delta}e^{-\frac{1}{2}(R_0^2 + \alpha' ln\frac{s}{M^2})q^2}\frac{1}{R_0^2 + \alpha' ln\frac{s}{M^2}}$$

$$\cdot ln[\frac{R_0^2 + 2\alpha' ln\frac{s}{M^2}}{R_0^2}] \tag{22}$$

We define

$$F_{PP}(s, q^2) = \frac{d\sigma}{dM^2}\frac{1}{4\pi} \tag{23}$$

$$\sigma_{tot}^{PP}(s) = 4\pi \, ImF_{PP}(s, q^2 = 0) \tag{24}$$

which is identical to the cross section orginally calculated by Streng [2].

The above cross section grows like $s^{2\Delta}$ with energy, much faster than σ_{tot}, so that s-channel unitarity corrections are necessary. We proceed to calculate these in a manner analogous to our previously published SD calculations [11]. For this purpose we write

$$\Omega_{PP}(s, b) = \frac{1}{8\pi\alpha'}\sigma_{PP}(M^2)g_0^4(\frac{s}{M^2})^{2\Delta}\frac{1}{R_0^2 + \alpha' ln\frac{s}{M^2}}ln[\frac{R_0^2 + 2\alpha' ln\frac{s}{M^2}}{R_0^2}]$$

$$\cdot \frac{1}{2\pi}\int d^2q e^{-i(q.b)}e^{-\frac{1}{2}(R_0^2 + \alpha' ln\frac{s}{M^2})q^2} \tag{25}$$

which can be rewritten as

$$\Omega_{PP}(s, b) = \frac{1}{8\pi\alpha'}ln[\frac{\bar{R}^2(\frac{s}{M^2})}{\bar{R}^2(1)}] \cdot \nu^2(\frac{s}{M^2}) \, e^{-\frac{2b^2}{R^2(\frac{s}{M^2})}} \tag{26}$$

The screened expression for $M^2\frac{d\sigma}{dM^2}$ is then

$$M^2\frac{d\sigma}{dM^2} = \frac{1}{8\pi\alpha'}\sigma_{PP}(M^2)ln[\frac{\bar{R}^2(\frac{s}{M^2})}{\bar{R}^2(1)}] \cdot \nu^2(\frac{s}{M^2})\int d^2b e^{-\Omega(s,b)}e^{-\frac{2b^2}{R^2(\frac{s}{M^2})}} \tag{27}$$

As we have already seen [11] this integral can be evaluated analytically yielding

$$M^2\frac{d\sigma}{dM^2} = \frac{1}{8\pi\alpha'}\sigma_{PP}(M^2)\nu^2(\frac{s}{M^2})ln[\frac{\bar{R}^2(\frac{s}{M^2})}{\bar{R}^2(1)}] \cdot a(\frac{1}{2\nu(s)})^a\gamma(a, 2\nu(s)) \tag{28}$$

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where a is defined as

$$a(s, M^2) = 2\,\frac{R^2(s)}{R^2(\frac{s}{M^2})} \geq 2 \tag{29}$$

This allows us to conclude that

$$< |D_{CD}|^2 > = a(\frac{1}{2\nu(s)})^a \gamma(a, 2\nu(s)) \xrightarrow[\alpha' lns \gg R_0^2]{} < |D_{SD}|^2 > \tag{30}$$

Although the formal structure of Eq.(30) appears to be identical to that of Eq. (9), there is a variance, as $a(s, M^2)$ is defined differently for SD and CD. For SD, $a \to 2$ from below, whereas for CD $a \to 2$ from above. A general mapping of the damping factor as a function of a and ν is presented in Fig.4.

The asymptotic energy dependence of the integrated σ_{CD} is presented in Table I. Once again we note that for present day energies $\nu \simeq 1$. Calculated values of $< |D_{CD}|^2 >$ are listed in Table II for some typical accelerator energies and are compared with the damping factors calculated for other processes. We have also included a calculation of $< |D_{CD}(m^2_{\chi(3415)})|^2 >$ for central $\chi_{c0}(3415)$ diffractive production where we have assumed $R_0^2(\chi(3415)) = 1 GeV^{-2}$.

Note that the above definition of $a(s, M^2)$, e.g Eq.(9), corresponds to the case where the projectiles survive the collision intact. For the case where projectiles are diffracted, $a(s, M^2)$ has a more complicated form, which does not converge as fast.

To summarize our conclusions: the main result derived from our calculations is that the energy scale at which the screening corrections for CD saturate, are approximately the same as that for SD [11]. However, the damping factors for CD are smaller than those for SD. As a result the uncorrected cross sections calculated for various CD channels have to be scaled down by a factor of 3 - 3.5 at Tevatron energies and by a factor of 5 in the LHC energy range. This differs dramatically from the damping calculated for the elastic amplitude.

Our second observation is that $< |D_i|^2 >$ can be factored out from the M^2 integration. The reason for this is, that $a(s, M^2)$ is almost a constant due to its logarithimic dependence on M^2. Consequently, our results (damping factors) are applicable to the various cross sections for DPE channels published recently [4-7]. These calculations were mainly made within the framework of the Low-Nussinov two gluon approximation of the Pomeron [9, 10]. Since, the amplitudes in this approximation appear to be well parametrized by an exponential in t-space, our method is applicable, and the amount of predicted damping can easily be deduce from Fig.4, or Table II. To be more specific we present in Fig.5 a comparison between the published calculations of the DPE contribution to heavy pair production [6] and our the results after damping corrections have been made.

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Figure Captions

Fig.1: The process of Double Pomeron Exchange (DPE).

Fig.2: A typical lego plot for DPE.

Fig.3: Relevant kinematics of our DPE calculation.

Fig.4: A mapping of damping factor as a function of $a(s, M^2)$ and $\nu(s)$.

Fig.5: DPE contribution to the cross section for heavy quark pair production versus the CM energy of the collision. Dashed lines denote the results of the uncorrected calculation of Bialas and Szeremeta [6]. Full lines are the results after screening corrections have been made.

Table Captions

Table I. Behaviour of the asymptotic cross section for uncorrected supercritical Pomeron model and the GLM model.

Table II. Damping factors at some typical accelarator energies

Table I.

	Supercritical Pomeron	GLM
σ_{tot}	s^{Δ}	$ln^2\frac{s}{s_0}$
σ_{el}	$\frac{s^{2\Delta}}{ln\frac{s}{s_0}}$	$ln^2\frac{s}{s_0}$
σ_{SD}	$\frac{s^{2\Delta}}{ln\frac{s}{s_0}}$	$ln\frac{s}{s_0}$
σ_{CD}	$\frac{s^{2\Delta}}{ln\frac{s}{s_0}}$	$ln\frac{s}{s_0}$
$\frac{\sigma_{el}}{\sigma_{tot}}$	$\frac{s^{\Delta}}{ln\frac{s}{s_0}}$	$\frac{1}{2}$
$\frac{\sigma_{diff}}{\sigma_{tot}}$	$\frac{s^{\Delta}}{ln\frac{s}{s_0}}$	$\frac{ln\frac{s}{s_0}}{s^{\Delta}}$

Table II.

$\sqrt{s}$ GeV	ν	$<|D_{tot}|^2>$	$<|D_{el}|^2>$	$<|D_{SD}|^2>$	$<|D_{CD}|^2>$	$<|D_{CD}(m^2_{\chi(3415)})|^2>$
180	0.76	0.838	0.626	0.446	0.344	0.356
540	0.86	0.820	0.592	0.402	0.337	0.328
1800	0.94	0.807	0.566	0.369	0.309	0.301
a = 2	1.00	0.797	0.548	0.296	0.296	0.296
8000	1.17	0.770	0.501	0.295	0.243	0.232
1400	1.20	0.765	0.494	0.289	0.234	0.223
40000	1.31	0.749	0.467	0.262	0.208	0.201

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